

# Sensor Repositioning to Improve Undersea Sensor Field Coverage

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**Abstract**—The growing demand for persistent underwater surveillance has led to a need to increase reliance on undersea distributed sensor networks for undersea target detection, classification and tracking. While tremendous progress has been made in the technology of small, relatively inexpensive sensors over the last decade, progress has lagged in the areas of sensor allocation and sensor management. How best to deploy and reposition sensors and small, unmanned vehicles (movable sensors) are important research questions that must be addressed to realize the intended use of these technologies.

Realistic tactical sensor deployment scenarios do not provide the opportunity for a precise placement of sensors. Most likely, initial deployment will be somewhat random (e.g., deployment of sensors from a moving vessel). Additionally, sensors might have to be repositioned due to random sensor failure, degradation, drift due to ocean current or other environmental effects. While it is possible, through the use of geometric probability, to estimate of the coverage of randomly distributed sensor fields, optimum field coverage can only be obtained through the use of deterministic sensor positioning procedures. However, the initial randomly distributed sensor field can be used as a starting point for the optimal sensor placement. The same can be said for networks in which sensors have drifted out of position, experienced failures, or have (through random movement or collision) aggregated into clumps. Sensor redeployment might also be necessary due to changes in mission objectives. For example, improved intelligence might necessitate the need to reconfigure the network in order to detect the target of interest.

This paper addresses various issues relating to repositioning of sensors in order to improve the coverage of the distributed sensor network. In addition to more traditional assignment algorithms, which minimize the total (equivalently, average) cost for moving all sensors, we consider various cost-based assignment techniques that aim to minimize maximal displacement. We argue that for some scenarios, especially small to moderate networks of sensors with limited fuel supply, the minimization of the maximal displacement is preferable to the solution of the more traditional assignment algorithm. The latter often produces results with relatively large costs for at least some of its assignments. This leads to diminished effectiveness over time for the sensor field. Since fuel supply is limited for these unmanned vehicles, we consider assignment procedures that will not deplete the vehicles' resources during the maneuvering phase. Finally, we compare the performance of several algorithms used to minimize the maximal cost associated with repositioning a field of movable sensors.

## I. INTRODUCTION

Realistic tactical sensor deployment scenarios do not provide the opportunity for a precise placement of sensors. Most likely, initial deployment will be somewhat random (e.g.,

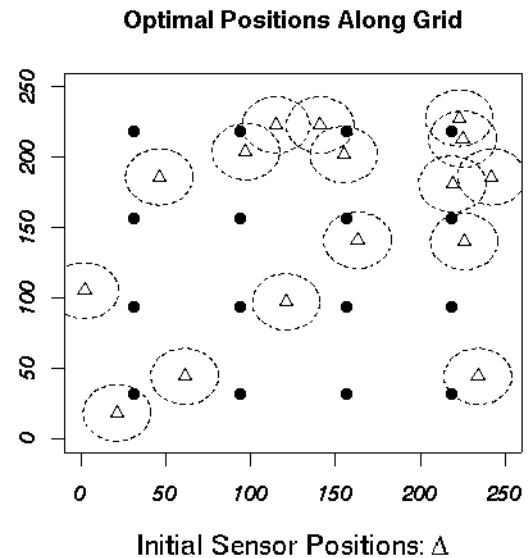


Fig. 1. Initial and desired sensor locations for a 16 sensor field

deployment of sensors from one or more moving vessels). For example, consider the scenario in Figure 1 in which the locations marked by the triangles represent the initial sensor positions. Notionally, the grid formed by the solid dots represents the desired (optimal) positions for the sensors.

As mentioned earlier, we can imagine that the sensors have been randomly scattered over the region by one or more crafts, or they may have drifted into these positions over time. It is also possible that there were originally several more sensors and that, due to random failure, these are all that are remaining. Here we use dashed circular lines to represent the hypothetical detection region about each sensor. In order to increase the coverage for the sensor field, the sensors must, in some way, be redeployed to the positions denoted by the black dots. Thus, our objective is to choose an assignment scheme which associates each sensor with a final location (i.e., one of the black grid locations).

Others have attempted to address this type of problem through the use of collaborative techniques [1], [2]. That is, each sensor continually adjusts its movement based on the

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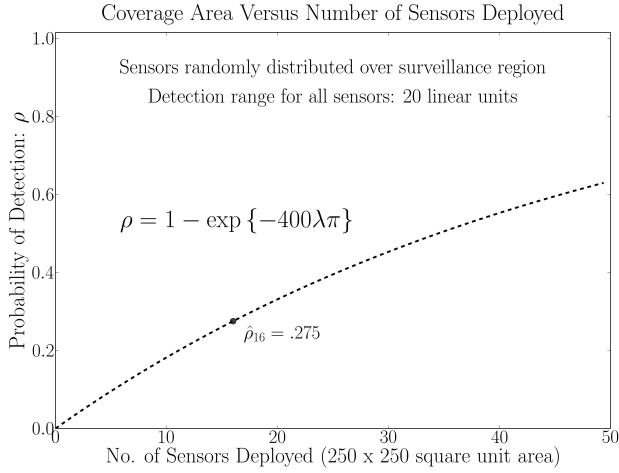


Fig. 2. Coverage probability for a  $250 \times 250$  area with  $r_0 = 20$

position of the other sensors. Thus, there is the assumption that the sensor positions are known at all times. Given that there is a cost overhead associated with communication, it seems reasonable then that we consider the option of determining the final locations for all sensors prior to the deployment of any of them. In this scenario, the sensors can move to their final destinations without expending energy in communicating during the repositioning procedure.

It is possible to estimate the coverage provided by a set of randomly distributed sensors over the surveillance region (c.f. [3]). For example, Figure 2 displays the expected coverage provided by a system of sensors randomly distributed over a 250 by 250 square unit area. In this example, each sensor has a detection range of twenty units. In general, the coverage provided by the randomly distributed system will be approximately  $\rho = 1 - \exp(-\lambda\pi r_0^2)$ , where  $\lambda$  is the sensor field intensity (i.e., the number of sensors deployed per unit area), and  $r_0$  is the detection range for the sensors.

The coverage is a function of sensor field intensity for the system of randomly distributed sensors. For this example, all sensors in the system have detection range of  $r_0 = 20$ . However, due to sensor overlap, it is clear that the randomly distributed system will not result in the optimal coverage alignment (see Figure 1). It is therefore necessary to reposition the sensors in order to improve system coverage.

## II. ASSIGNMENTS

### A. An Assignment Procedure

Sensors have limited energy supply. Thus energy consumed during repositioning will no longer be available for surveillance purposes. One should therefore consider assignment schemes that, in some sense, minimize energy consumption. For example, an assignment algorithm [4] can be used as a baseline method to determine how to associate the sensors with the optimum positions. One variant of the standard assignment problem minimizes the total cost for the overall system assignments:

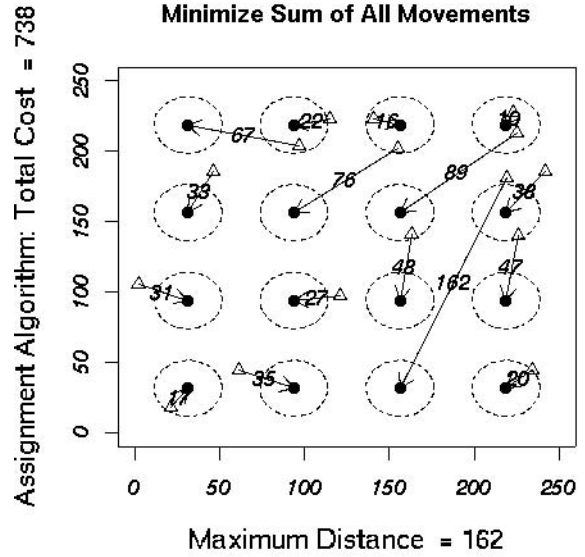


Fig. 3. Sensor assignment with a conventional assignment algorithm

### Assignment Optimization Problem

$$\begin{aligned}
 &\text{minimize} && \sum_{j=1}^n \sum_{i=1}^n c_{i,j} x_{i,j} \\
 &\text{subject to} && \sum_{j=1}^n x_{i,j} = 1 \text{ for } i = 1, 2, \dots, n \\
 &&& \sum_{i=1}^n x_{i,j} = 1 \text{ for } j = 1, 2, \dots, n \\
 &&& x_{i,j} \in \{0, 1\}, \text{ for all } i, j
 \end{aligned}$$

Where  $c_{i,j} = f(D_{i,j})$ ,  $f(D)$  is a non-negative and non-decreasing function that represents the cost of moving distance  $D$ , and  $D_{i,j}$  is the distance from sensor  $i$  to position  $j$ . In this assignment problem a value of  $x_{i,j} = 1$  corresponds to sensor  $i$  being associated with (equivalently, “moving to”) final position  $j$ . The first constraint guarantees that each sensor can only move to a single final position; and the second constraint guarantees that each final position is occupied by exactly one (and only one) sensor. A key feature of the assignment optimization problem shown above is that it can be solved as a purely linear program (as opposed to an integer program) and the optimal solution is guaranteed to be the integer result. The conversion to a linear program is accomplished by replacing the  $x_{i,j} \in \{0, 1\}$  constraint with  $x_{i,j} \geq 0$ . This well-known result (given in reference [4] as Theorem 8.5) leads to many computationally robust assignment algorithms, since efficient simplex-type algorithms can be applied to this problem to yield the optimal integer result.

In Figure 3, we show the resulting assignment paths from solving the standard assignment optimization problem for the example from Figure 1. In this example, the overall system cost of moving the sensors to the desired locations is 738

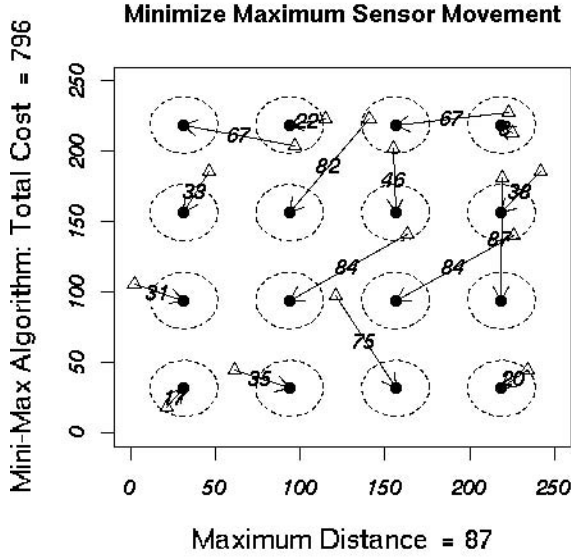


Fig. 4. Optimal minimax sensor assignments

units. However, notice that one sensor has incurred a cost of 162 units (22 percent of the overall system cost). This is not uncommon with traditional assignment algorithms: In general, the closest possible associations are made; however, a few associations will be relatively costly. Because of limited energy available to the sensors, the resulting poor performance of sensors that have moved great distances is likely to diminish the effectiveness of the network. It is thus more desirable to achieve the assignment without these long paths whenever feasible.

#### B. Minimizing Maximal Cost

We solve the problem of reducing long travel paths by using a modified assignment scheme which minimizes the maximum displacement over all sensors. The idea is to assign the sensors to the locations in such a way as to obtain more equitable energy consumption. For example, for the same initial distribution of sensors as in Figure 1, using a minimax assignment algorithm reduces the maximal energy consumption from 162 to 87 (a reduction of 46 percent). In achieving this reduction, the overall system cost (sum of paths) increased only 7.86 percent from 738 to 796. The resulting minimax assignment paths for the example from Figure 1 are shown in Figure 4. The minimax improvement is achieved by reducing the long path (length of 162 in Figure 3) from the traditional assignment result. However, to meet the one-sensor-to-one-position constraints, the algorithm must shuffle most of the other paths. This gives the effect of the minor increase in the sum paths with the major reduction of the maximal path. However, this also illustrates that a simple modification of the standard assignment result is not an effective approach to reducing large paths. A formal method to directly solve the minimax assignment problem is thus required.

The minimax assignment problem is stated in optimization form as follows:

#### Minimax Assignment Optimization Problem

$$\begin{aligned}
 &\text{minimize} && \text{maximum } c_{i,j}x_{i,j} \\
 &\text{subject to} && \sum_{j=1}^n x_{i,j} = 1 \text{ for } i = 1, 2, \dots, n \\
 &&& \sum_{i=1}^n x_{i,j} = 1 \text{ for } j = 1, 2, \dots, n \\
 &&& x_{i,j} \in \{0, 1\}, \text{ for all } i, j
 \end{aligned}$$

where, as before,  $c_{i,j} = f(D_{i,j})$ ,  $f(D)$  is a non-negative and non-decreasing function that represents the cost of moving distance  $D$ , and  $D_{i,j}$  is the distance from sensor  $i$  to position  $j$ . Once again,  $x_{i,j} = 1$  if sensor  $i$  is associated with final position  $j$ , and  $x_{i,j} = 0$  otherwise. The conversion from this integer constraint to a linear constraint (as in the conventional assignment problem) is not guaranteed to create an equivalent optimal integer solution in this case; thus, the minimax assignment problem requires more computationally complex algorithms.

#### C. Minimax Assignment Algorithms

We propose the following algorithm to solve the minimax assignment problem for sensor path selection:

- 1) Use the traditional assignment algorithm to make the initial sensor-location associations.
- 2) Save the current solution (call it *potential-sol*) and remove the link associated with the maximum cost from the network: this can be done, for example, by assigning it a very large value.
- 3) Apply the Assignment Algorithm to the modified network.
  - (a) If a solution exists whose maximum cost is less than that of the *potential-sol*, go back to step 2.
  - (b) Otherwise, stop; *potential-sol* is optimal.

Notice that, unlike the traditional (minisum) assignment algorithm, this minimax assignment algorithm is iterative. However, the estimates for the iterations monotonically converge to an optimal solution. The estimates converge to an optimal solution because at each step we remove a link (the maximal link for the previous iteration) no longer than the link previously removed. The program stops only when the removal of a link results only in solutions containing a maximal link that is larger than the last link removed from the network. Thus, if  $D_M$  is the maximal displacement of an optimal solution, then eventually the algorithm will achieve a solution with  $D_M$  as the length of its maximal link. This algorithm was used to create the solution shown in Figure 4. The minimax assignment algorithm is a variant of one developed by Corley and Golnabi [5], the primary difference being that we update only one cost per iteration. In [5], all links of value equal to the current maximum are effectively removed from the network. Our simulations show that, depending on the

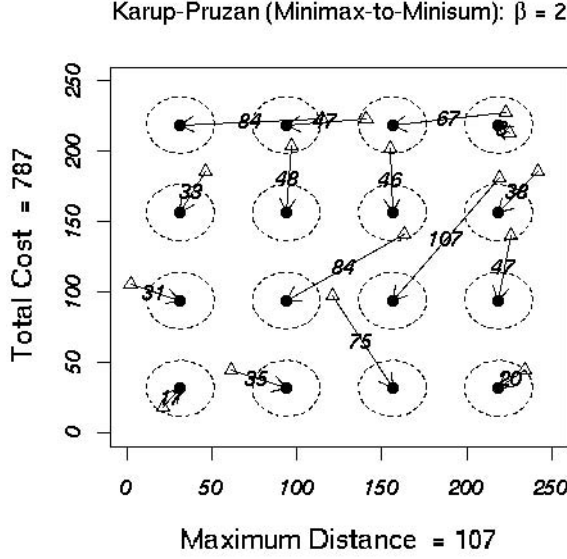


Fig. 5. Minimax sensor assignments obtained using Krarup and Pruzan's algorithm with  $\beta = 2$

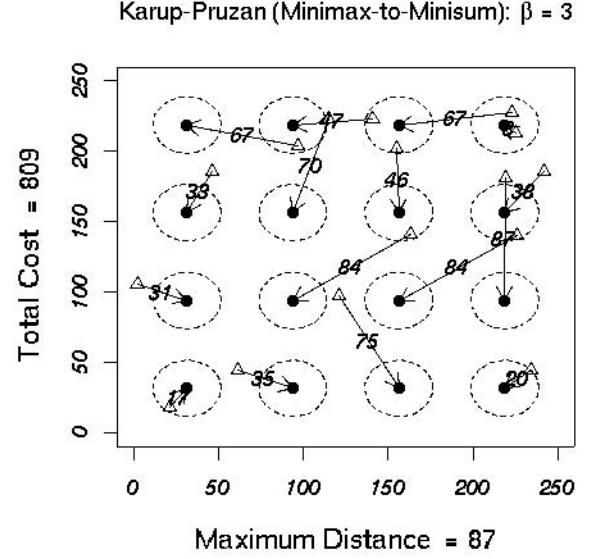


Fig. 6. Minimax sensor assignments obtained using Krarup and Pruzan's algorithm with  $\beta = 3$

underlying assignment routine, the removal of multiple links at once can lead to a significant increase in execution time.

### III. REDUCIBILITY OF MINIMAX TO MINISUM

Krarup and Pruzan [6] suggest a simple transformation for reducing certain minimax integer programming problems to minisum problems. Essentially, the idea amounts to transforming the data in such a way as to exaggerate the contribution of the larger network links. Thus, if  $\mathbf{T}$  is such a transformation, then

$$\begin{aligned} c_{i,j} > c_{r,p} &\implies \mathbf{T}(c_{i,j}) \gg \mathbf{T}(c_{r,p}) \\ c_{i,j} = c_{r,p} &\implies \mathbf{T}(c_{i,j}) = \mathbf{T}(c_{r,p}) \end{aligned}$$

where the symbol  $\gg$  means *far greater than*. The modified cost matrix is then used in some minisum assignment procedure. For example, Krarup and Pruzan suggested using the transformation  $\mathbf{T}[x] = x^\beta$ ,  $\beta > 1$ . Applying this transformation to the cost matrix used earlier leads to the result shown in Figure 5. This suggests that raising all terms to an even higher power (and thereby magnifying the spread of the distances) might lead us the optimum solution. In Figure 6 we show the result of applying the algorithm of Krarup and Pruzan with  $\beta = 3$ . Note however that the total system cost is larger than that obtained by our modified Corley-Golnabi procedure (as shown in Figure 4). This remains the case even if we further increase the power of the cost terms, as shown in Figure 7 for  $\beta = 7$ .

Jorgensen and Powell [7] suggested a modification of the method of Krarup and Pruzan which ensures that the transformation of each cost is greater than the sum of the transformations of all lesser cost terms. The procedure first requires a re-indexing of the cost terms such that  $c_1 \leq c_2 \leq \dots \leq c_n$ ,

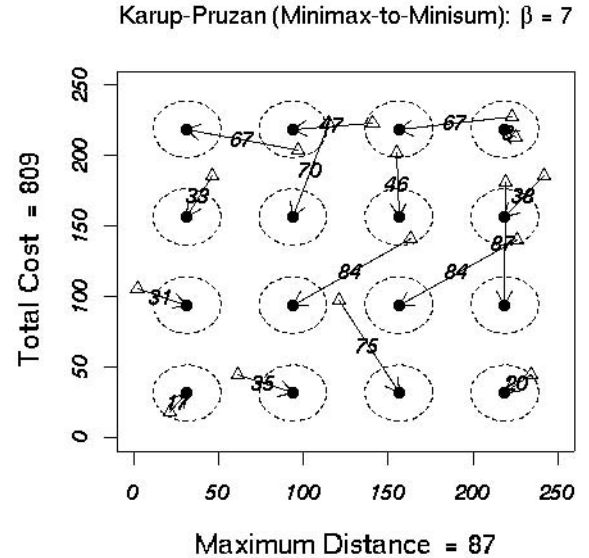


Fig. 7. Minimax sensor assignments obtained using Krarup and Pruzan's algorithm with  $\beta = 7$

and setting  $\mathbf{T}[c_1] = 0$ . The remaining transformations are obtained via:  $c_j > c_{j-1} \implies \mathbf{T}(c_j) = 1 + \sum_{i=1}^{j-1} \mathbf{T}(c_i)$  and  $c_j = c_{j-1} \implies \mathbf{T}(c_j) = \mathbf{T}(c_{j-1})$ . For example, the cost values  $c = (4, 7, 16, 19, 34, 36)$  are transformed into  $(0, 1, 2, 4, 8, 16)$ . Note the powers of two for all but the first term for the transformed data. This is always the case when the cost values are unique and can be problematic for large systems. For example, it produces the estimate shown in Figure 8 for the minimax assignment example of Figure 4.

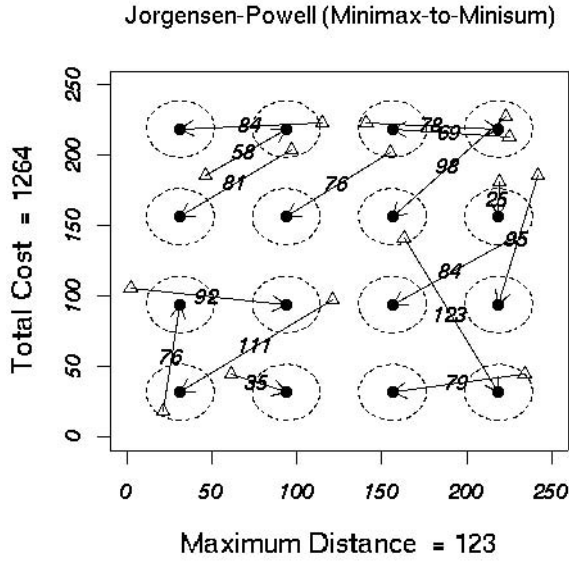


Fig. 8. Minimax sensor assignments obtained using Jorgensen and Powell's algorithm

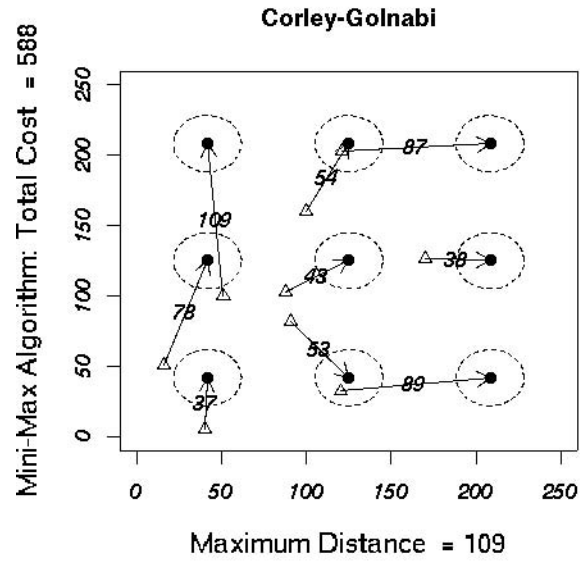


Fig. 10. Optimal minimax assignments for the field in Figure 9

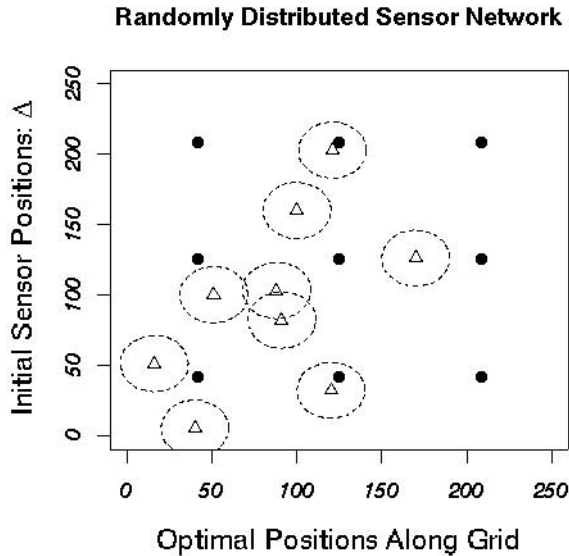


Fig. 9. Initial and desired sensor locations for a 9 sensor field

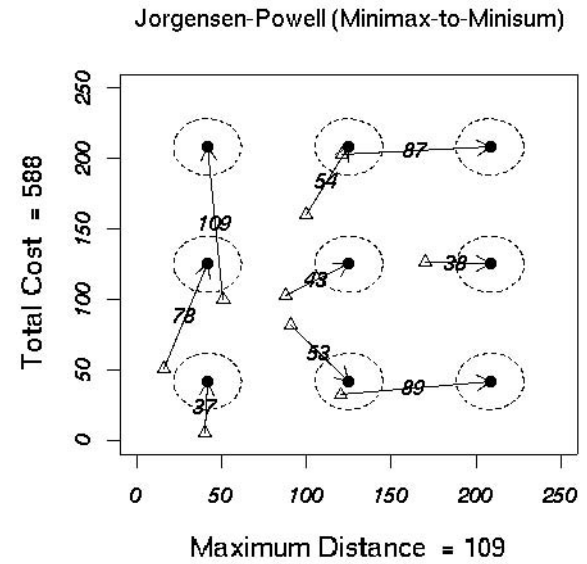


Fig. 11. Minimax sensor assignments obtained using Jorgensen and Powell's algorithm for the field in Figure 9

Before trying to understand why the resulting maximal distance from Jorgensen and Powell is so much larger than those obtained from other algorithms, let us first consider a smaller problem. We consider a distribution of 9 sensors with starting and desired locations as shown in Figure 9. The resulting optimal minimax assignment (as determined by our modified Corley-Golnabi method) is shown in Figure 10. In Figure 11, we show the result of apply the algorithm of Jorgensen and Powell to this smaller field of nine sensors. Based on the match between Figures 10 and 11 it is clear that

the transformation of Jorgensen and Powell does lead to the correct solution for this small system. However, as mentioned earlier, if all the distances between the sensors and the potential locations are unique, then the transformations lead to the powers of two:  $T[c_2] = 1, T[c_2] = 2, T[c_3] = 4, \dots$ . A system of sixteen sensors and sixteen possible locations for each sensor, has 256 unique sensor-location pairings. Thus, we can expect significant rounding to occur for problems of this type<sup>1</sup> and, as demonstrated above, the estimates can be

<sup>1</sup>For example,  $T[c_{256}] = 2^{254}$ .

far from optimum. Thus, we use our modified Corley-Golnabi algorithm to determine appropriate minimax paths in practice.

#### IV. CONCLUSION

This paper examines the problem of redistributing members from a network of mobile unmanned sensors in order to improve system-wide performance. It is assumed that energy is consumed while the sensors migrate to their new locations, thereby reducing the performance of the reconfigured network. We considered various algorithms to determine sensor placement while minimizing the maximal displacement required for any individual sensor in the system. It is shown that a modified version of the Corley-Golnabi algorithm is optimal in the sense that it minimizes the maximal distance travelled, while simultaneously providing a nearly minimal total cost (sum distance) for the system. Future work will address the collaborative movement of the sensors in order to maintain coverage over time.

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